32. \[ \{1, 3, 5\} \oplus \{1, 2, 3\} = \{2, 5\} \]

34. \( A \oplus B \):

\[
\begin{array}{cccccc}
A & | & B & | & A \oplus B & | & A - B & | & B - A & | & (A - B) \cup (B - A) \\
0 & | & 0 & | & 0 & | & 0 & | & 0 & | & 0 \\
0 & | & 1 & | & 1 & | & 0 & | & 1 & | & 1 \\
1 & | & 0 & | & 1 & | & 1 & | & 0 & | & 1 \\
1 & | & 1 & | & 0 & | & 0 & | & 0 & | & 0 \\
\end{array}
\]

Since the third and last columns are the same, we see that \( A \oplus B = (A - B) \cup (B - A) \).

36. We’ll use a membership table:

38. There are various ways to do this problem. I’ll use the previous problem, which says that

\( A \oplus B = (A - B) \cup (B - A) \).

We also know that \( \cup \) is commutative, \( i.e. \ X \cup Y = Y \cup X \) for any sets \( X \) and \( Y \). Thus,

\( (A - B) \cup (B - A) = (B - A) \cup (A - B) \).

But now by the previous problem again, we know that

\( (B - A) \cup (A - B) = B \oplus A \).

Putting it all together, we see that

\( A \oplus B = B \oplus A \),

as desired.

Section 2.3
2. (a) No, because $n$ does not map to a unique element of $\mathbb{R}$.
(b) Yes and no, depending on how we interpret $\sqrt{x}$. If, as is usually assumed, $\sqrt{x}$ means take the positive square root of $x$, then yes, $\sqrt{n^2 + 1}$ is a function from $\mathbb{N}$ to $\mathbb{R}$. However, if $\sqrt{x}$ means take any square root, then we have a sign ambiguity, so $\sqrt{n^2 + 1}$ is not a function.
(c) No, because $n = 2$ cannot possibly be in the domain of the function as the number $1/0$ is not a real number.

4. (a) Domain: $\mathbb{N}$. Range: $\{x \in \mathbb{Z} \mid x > 0\}$.
(b) Domain: $\{x \in \mathbb{Z} \mid x > 0\}$. Range: $\{x \in \mathbb{Z} \mid x > 1\}$.
(c) Domain: $\bigcup_{k \geq 0} \{0, 1\}^k$. Range: $\mathbb{N}$.
(d) Domain: $\bigcup_{k \geq 0} \{0, 1\}^k$. Range: $\{x \in \mathbb{Z} \mid x > 1\}$.

14. (a) Yes
(b) No, for example, because the number 2 is not in the range: there do not exist integers $m$ and $n$ such that $m^2 - n^2 = (m - n)(m + n) = 2$.
(c) Yes
(d) Yes
(e) No

20. (a) The function
\[
   f_a : \mathbb{N} \to \mathbb{N}
   \quad n \mapsto n + 1
\]
is one-to-one, but not onto.
(b) The function
\[
   f_b : \mathbb{N} \to \mathbb{N}
   \quad \text{defined by}
   \begin{cases}
      0 & \text{if } n = 0 \\
      n - 1 & \text{otherwise}
   \end{cases}
\]
is onto, but not one-to-one.
(c) The function
\[
   f_c : \mathbb{N} \to \mathbb{N}
   \quad \text{defined by}
   \begin{cases}
      1 & \text{if } n = 0 \\
      0 & \text{if } n = 1 \\
      n & \text{otherwise}
   \end{cases}
\]
is onto and one-to-one (but not the identity function).
(d) The constant function
\[
   f_d : \mathbb{N} \to \mathbb{N}
   \quad \text{defined by}
   f_d(n) = 0
\]
is neither one-to-one nor onto.
34. Yes. Suppose this were not the case. Then there would exist nonempty sets $A, B$ and $C$, and functions $f : B \to C$ and $g : A \to B$ such that $f$ and $f \circ g$ are one-to-one, but $g$ is not one-to-one. Since $g$ is not one-to-one, this means there exist two elements $x, y \in A$ such that $x \neq y$ but $g(x) = g(y)$ in $B$. But then that implies
\[ f \circ g(x) = f(g(x)) = f(g(y)) = f \circ g(y), \]
which contradicts the fact that $f \circ g$ is one-to-one.

38. Note that
\[ f \circ g(x) = a(cx + d) + b \]
whereas
\[ g \circ f(x) = c(ax + b) + d. \]
If we want to know when $f \circ g = g \circ f$, we need to understand when
\[ a(cx + d) + b = c(ax + b) + d. \]
This equation is equivalent to
\[ acx + ad + b = acx + bc + d. \]
Thus, $f \circ g = g \circ f$ precisely when
\[ ad + b = bc + d. \]